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Nonlinear theory of electromagnetic wave excitation by a relativistic electron beam in a metal waveguide, filled with a dielectric medium

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Abstract. The problem of nonlinear interaction of a relativistic electron beam with a bounded dielectric medium, which fills the metal waveguide is considered. General formulae for the frequency spectra and growth rates of the beam-excited electromagnetic waves as well as the expression for the threshold-wave amplitudes in the nonlinear stage of generation are obtained. The electromagnetic radiation energy flow and the efficiency of generation for values close to the threshold saturation are calculated.

1. Introduction

A large number of papers, dedicated to the investigation of the nonlinear collective interaction of relativistic electron beams with plasmas, have recently been published. A review of these investigations, of which a major part is concerned with space unbound beam–plasma systems, has been given lately (Dolgenko *et al* 1974). In this review the problem of the electromagnetic wave excitation upon relativistic electron beam action in an unbound dielectric medium with a permittivity $\epsilon_0(\omega)$ was solved. The real system however, in which electron beams are used for the generation of coherent microwave radiation, is seen to be space limited. This is the reason why a consistent nonlinear theory of electron beam interaction with a bounded dielectric medium is of interest and its development is the purpose of this paper.

It must be noted that the linear theory of electromagnetic wave excitation in bounded beam–plasma systems is fully developed (Aranov *et al* 1974). As far as nonlinear theory is concerned, a single paper is known (Tchogovadze 1973). In this paper the interaction of the relativistic beam with the longitudinal (potential) wave of a finite amplitude in a plasma cylinder, bounded by a metal waveguide, is studied. Further, the results from this paper (Tchogovadze 1973) can be obtained from the more general formulae about the arbitrary dielectric medium (with $\epsilon_0(\omega)$), filling a waveguide.

The electromagnetic wave in a guide is a travelling wave only along the guide axis, whereas it has a complex structure radially, the structure being similar to a non-homogeneous standing wave. When such a wave with a finite amplitude interacts with an electron beam, the radial-averaged Miller's force (Gaponov and Miller 1958, Gorbunov 1973) must be kept in mind and this force predetermines that the beam must move radially. The action of this force will be further neglected, since it is assumed that there is a constant magnetic field $\boldsymbol{B}_0$ along the axis and this field is sufficiently strong to meet the condition

$$\Omega_e \gg \omega_B, \tag{1.1}$$

where $\Omega_e = eB_0/mc$ is the cyclotron frequency of the electrons in the field $\boldsymbol{B}_0$ and $\omega_B = 4\pi e^2 n_0/m$ is Langmuir's frequency of beam-electrons with density in the laboratory coordinate system.

In such a magnetic field the transverse motion of beam electrons (radial and azimuthal) can be fully neglected. As far as the longitudinal motion is concerned, the velocity of which in the absence of electromagnetic waves is equal to $\boldsymbol{u} = (0, 0, u)$, this motion is changed by the effect of the wave field. Since our interest lies in the beam-excited electromagnetic waves, for which the resonance condition of radiation must hold ($\omega = k_z u$), even a small deviation from the longitudinal electron velocity may cause a great change in the character of the wave propagation in the system under consideration.

Thus the nonlinear action of the excited electromagnetic wave is taken into account only for the longitudinal motion of the beam electrons. The dielectric medium is considered to be linear and is described by the tensor of the dielectric permittivity:

$$\epsilon_{0ij} = \begin{pmatrix} \epsilon_\perp(\omega) & 0 & 0 \\ 0 & \epsilon_\perp(\omega) & 0 \\ 0 & 0 & \epsilon_\parallel(\omega) \end{pmatrix}. \tag{1.2}$$

Two possible cases are studied.

(i) An isotropic dielectric medium, for which $\epsilon_\perp = \epsilon_\parallel = \epsilon_0(\omega)$ (this is the case, for instance, with a plasma in the frequency range $\omega \gg \Omega_e$, the permittivity being $\epsilon_0 = 1-(\omega_p^2/\omega^2)$, where ω_p is Langmuir's plasma electron frequency).

(ii) A single-axis anisotropic dielectric with $\epsilon_\perp = 1$ and $\epsilon_\parallel = \epsilon_0(\omega)$ (this case is realized in a strong magnetized plasma in the frequency range $\omega < \Omega_e$ and when the condition $\omega_p \ll \Omega_e$ is satisfied).

The beam electron motion can be described using the equation of relativistic hydrodynamics (Lovetskii and Rukhadze 1965)

$$\begin{aligned} &\frac{\partial n}{\partial t} + \operatorname{div} n\boldsymbol{v} = 0 \\ &\left(\frac{\partial}{\partial t} + \boldsymbol{v}\nabla\right)\frac{\boldsymbol{v}}{[1-(v^2/c^2)]^{1/2}} = \frac{e}{m}\left(\boldsymbol{E} + \frac{1}{c}(\boldsymbol{v}\times\boldsymbol{B})\right) \end{aligned} \tag{1.3}$$

where n is the density and $\boldsymbol{v}$ the velocity of the beam electrons. The electric and magnetic fields are determined by the self-consistent system of Maxwell's equations:

$$\begin{aligned} &\operatorname{div}\mathscr{D} = 4\pi e(n-n_0), &&\operatorname{curl}\boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t}, \\ &\operatorname{curl}\boldsymbol{B} = \frac{1}{c}\frac{\partial \mathscr{D}}{\partial t} + \frac{4\pi}{c}e(n\boldsymbol{v} - n_0\boldsymbol{u}), &&\operatorname{div}\boldsymbol{B} = 0. \end{aligned} \tag{1.4}$$

Here $\rho_0 = en_0$ and $\boldsymbol{j}_0 = en_0\boldsymbol{u}$ are the unperturbed charge density and the beam-current density, respectively and $\mathscr{D}_i = \epsilon_{0ij}(\omega)E_j$, which characterizes the electrodynamic properties of the dielectric medium, filling the metal guide, with a radius R.

The equations (1.3) and (1.4) with the boundary conditions on the metal-guide surface

$$E_z|_{r=R} = E_\varphi|_{r=R} = 0 \tag{1.5}$$

represent a complete system of equations for the problem under consideration, namely, the nonlinear electromagnetic wave excitation by an electron beam in a dielectric medium bounded by a metal guide.

2. Linear theory of electromagnetic wave excitation in the system considered

Before undertaking the analysis of the nonlinear problem, we shall give here the basic results of the linear theory of electromagnetic wave excitation by a relativistic electron beam in the system under consideration. If the equations (1.3) and (1.4) are linearized and if time and position dependences for non-equilibrium quantities are assumed in the form

$$f(r)\exp(-\mathrm{i}\omega t+\mathrm{i}l\varphi+\mathrm{i}k_z z), \tag{2.1}$$

the permittivity tensor of a system, consisting of a dielectric medium and a magnetized electron beam, is found:

$$\epsilon_{ij} = \begin{pmatrix} \epsilon_1 & 0 & 0 \\ 0 & \epsilon_1 & 0 \\ 0 & 0 & \epsilon_3 \end{pmatrix}, \tag{2.2}$$

where

$$\epsilon_1 = \epsilon_0(\omega), \qquad \epsilon_3 = \epsilon_0(\omega)-\frac{\omega_{\mathrm{B}}^2\gamma^{-3}}{(\omega-k_z u)^2} \tag{2.3}$$

for the case of an isotropic medium and

$$\epsilon_1 = 1, \qquad \epsilon_3 = \epsilon_0(\omega)-\frac{\omega_{\mathrm{B}}^2\gamma^{-3}}{(\omega-k_z u)^2} \tag{2.4}$$

for an anisotropic single-axis dielectric, and further $\gamma = [1-(u^2/c^2)]^{-1/2}$.

Using the tensor (2.2) the field equations can be reduced to two equations for the components B_z and E_z (B and E waves, respectively)

$$\Delta_\perp B_z-\kappa^2 B_z = 0, \tag{2.5a}$$

$$\Delta_\perp E_z-\frac{\epsilon_3}{\epsilon_1}\kappa^2 E_z = 0 \tag{2.5b}$$

where

$$\kappa^2 = k_z^2-\epsilon_1\frac{\omega^2}{c^2} \qquad \text{and} \qquad \Delta_\perp = \frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r}-\frac{l^2}{r^2}.$$

The boundary conditions (1.5) for these equations take the form

$$E_z|_{r=R} = \left.\frac{\partial B_z}{\partial r}\right|_{r=R} = 0. \tag{2.6}$$

The boundary-value problems thus formulated have the following solution:

$$B_z = B_{z0}J_l(i\kappa r) \qquad E_z = E_{z0}J_l\left(i\left(\frac{\epsilon_3}{\epsilon_1}\kappa^2\right)^{1/2} r\right), \tag{2.7}$$

while the eigenvalues spectrum ω is found from the dispersion equations:

$$k_z - \epsilon_1\frac{\omega^2}{c^2} + \frac{\mu_{sl}'^2}{R^2} = 0 \tag{2.8a}$$

$$\frac{\epsilon_3}{\epsilon_1}k_z^2 - \epsilon_3\frac{\omega^2}{c^2} + \frac{\mu_{sl}^2}{R^2} = 0, \tag{2.8b}$$

where μ_{sl} and μ_{sl}' are the roots of a Bessel function and its derivative ($J_l(\mu_{sl}) = 0$, $J_l'(\mu_{sl}') = 0$).

Now, knowing E_z and B_z the other components of the electric and magnetic fields of the wave are easily found:

$$\begin{aligned}
E_\varphi &= \frac{1}{\kappa^2}\left(\frac{i\omega}{c}\frac{\partial B_z}{\partial r} + \frac{l}{r}k_z E_z\right),\\
E_r &= \frac{1}{\kappa^2}\left(\frac{l}{r}\frac{\omega}{c}B_z - ik_z\frac{\partial E_z}{\partial r}\right),\\
B_\varphi &= \frac{1}{\kappa^2}\left(\frac{l}{r}k_z B_z - \frac{i\omega}{c}\epsilon_1\frac{\partial E_z}{\partial r}\right),\\
B_r &= \frac{1}{\kappa^2}\left(ik_z\frac{\partial B_z}{\partial r} + \frac{l\omega}{rc}\epsilon_1 E_z\right).
\end{aligned} \tag{2.9}$$

It follows from (2.8a) that the B wave is always stable; the beam does not interact with this wave. This is the reason why it is further postulated that $B_z = 0$. With respect to the E wave, which is described by (2.8b), it is excited by the beam and further the beam interacts with the wave. Here

$$\omega = \omega_0 + i\delta = k_z u + i\delta, \tag{2.10}$$

where ω_0 is the frequency of the wave excited by a beam and δ characterizes the growth rate.

In the case of an isotropic dielectric medium, the excitation of both the almost-longitudinal (potential) and the almost-transverse electromagnetic waves is possible. In the frequency range, in which $\epsilon_0(\omega) < 1$, the almost-longitudinal waves are excited by the beam and the spectrum of these waves is determined by the equation

$$\epsilon_0(\omega_0) = 0 \tag{2.11}$$

and the growth rate is given by the expression:

$$\delta = \frac{-i+\sqrt{3}}{2\gamma}\left(\frac{\omega_B^2}{[1+(\mu_{sl}^2u^2/R^2\omega_0^2)]\partial\epsilon_0(\omega_0)/\partial\omega_0}\right)^{1/3}. \tag{2.12}$$

In the frequency range, in which $\epsilon_0(\omega_0) > 1$, the almost-transverse electromagnetic waves are excited by the beam and the waves are in the frequency spectrum

$$\frac{\mu_{sl}^2}{R^2} = \frac{\omega_0^2}{u^2}\left(\frac{u^2}{c^2}\epsilon_0(\omega_0) - 1\right), \tag{2.13}$$

the growth rate being

$$\delta = \frac{-\mathrm{i}+\sqrt{3}}{2\gamma}\left(\frac{c^2\mu_{sl}^2\omega_{\mathrm{B}}^2}{R^2\epsilon_0(\omega_0)(\partial/\partial\omega_0)[\omega_0^2\epsilon_0(\omega_0)]}\right)^{1/3}. \tag{2.14}$$

It follows from the formulae (2.11)–(2.14), that when the almost-longitudinal wave is excited by the beam in an isotropic dielectric medium, the fundamental axial symmetrical mode with $s = 0$ and $l = 0$ ($\mu_{00} = 2{\cdot}4$) has a maximum growth rate. As far as the transverse electromagnetic wave is concerned, the problem is more complicated. At $\epsilon_0(\omega) =$ constant the growth rate increases, when the number of modes is increased, as $\mu_{sl}^{1/3}$. In practice, such is the increase in the rate with the number of modes at an arbitrary $\epsilon_0(\omega_0) > 1$. That is why, if no special steps are taken, the excitation of the transverse wave must be non-coherent and multimode in character. But if the beam is slightly modulated in advance one mode generation is always possible (Fainberg 1968).

Now let us consider the wave excitation in the case of an anisotropic single-axis dielectric. It can easily be shown that in this case the electromagnetic wave excitation is possible only in the frequency range, for which $\epsilon_0(\omega_0) < 0$. The frequency spectrum of oscillations is determined by the equation

$$\omega_0^2\epsilon_0(\omega_0)+\gamma^2\frac{\mu_{sl}^2u^2}{R^2} = 0 \tag{2.15}$$

and the growth rate is

$$\delta = \frac{-\mathrm{i}+\sqrt{3}}{2\gamma}\left(\frac{\omega_0^2\omega_{\mathrm{B}}^2}{\omega_0^2(\partial\epsilon_0(\omega_0)/\partial\omega_0)+(2\mu_{sl}^2u^2/\omega_0R^2)\gamma^2(\gamma^2-1)}\right)^{1/3}. \tag{2.16}$$

When the electron beam is non-relativistic, that is $\gamma \to 1$, the phase velocity of the waves considered is smaller than the light velocity and the wave field proves to be a potential field and this holds with a high enough precision.

From formulae (2.15) and (2.16) for strongly magnetized plasma $\epsilon_0(\omega) = 1-(\omega_0^2/\omega^2)$ it follows, that an excitation of the oscillation is possible only provided that

$$\omega_{\mathrm{p}}^2 > \mu_{sl}^2u^2\gamma^2R^{-2}$$

(Aronov *et al* 1974). This is the reason why if the plasma density lies within the range

$$\left(\frac{3{\cdot}8u\gamma}{R}\right)^2 > \omega_{\mathrm{p}}^2 > \left(\frac{2{\cdot}4u\gamma}{R}\right)^2, \tag{2.17}$$

the beam will excite one axial symmetrical mode with $\mu_{00} = 2{\cdot}4$.

In conclusion it should be noted that the instability considered above leads to an excitation of a wave, for which $E_z \neq 0$, and that is why the work of the field upon the electron beam is nonzero, $\boldsymbol{E}\,.\,\boldsymbol{u} \neq 0$. Therefore the wave excitation must be accompanied by a beam deceleration, which in turn invalidates the resonance condition $\omega_0 = k_z u$ and the instability becomes stable.

3. Nonlinear saturation of a beam-excited electromagnetic wave in a dielectric

It has been proved above that the small amplitude electromagnetic waves, described by the linear theory, in the system under consideration are unstable; the amplitude of these waves increases with time upon the action of the beam. At a sufficiently large amplitude, however, the instability must tend to saturation. As noted elsewhere (Dolgenko *et al* 1974) the waves for sufficiently large amplitudes, being capable of breaking the resonance condition, cannot increase in amplitude and they become stationary. Let us calculate the threshold amplitude; above this threshold amplitude the beam-excited wave is stationary.

The electromagnetic wave will be considered as stationary and a variable $\xi = t-(k_z/\omega)Z$ will be introduced. All quantities are dependent only on this variable (in addition to the dependences on r and φ). For the case of a magnetized beam, when the electron motion is one-dimensional, the following integrals in the equations (1.3) are found (Dolgenko *et al* 1974):

$$n = n_0\frac{k_z u-\omega}{k_z v_z-\omega},$$

$$\frac{k_z c}{\omega}(m^2c^2-P_z)^{1/2}-P_z+\frac{ek_z}{\omega}\phi = \frac{k_z c}{\omega}mc\gamma\left(1-\frac{u\omega}{k_z c^2}\right), \tag{3.1}$$

where

$$P_z^2 = \frac{v_z^2}{c^2}(m^2c^2+p_z^2).$$

The function ϕ is connected with E_z in the following way:

$$E_z = \frac{k_z}{\omega}\frac{\partial\phi}{\partial\xi}. \tag{3.2}$$

All other components of the electric and magnetic fields E_z and B_z can be found from the equations (2.9) (taking into account that $B_z = 0$).

Using (3.1) and Maxwell's equations a nonlinear equation for ϕ is obtained,

$$\epsilon_0(\omega)\left(\Delta_\perp\phi+\frac{\kappa^2}{\omega^2}\frac{\partial^2\phi}{\partial\xi^2}\right) = \omega_B^2\frac{m\kappa_0^2}{ck_z^2}\left(\frac{u-(\omega/k_z)}{\{[u-(\omega/k_z)]^2-(2e\phi/m\gamma^3)\}^{1/2}}-1\right) \tag{3.3}$$

for the case of an isotropic dielectric and

$$\Delta_\perp\phi+\frac{\epsilon_0(\omega)\kappa_0^2}{\omega^2}\frac{\partial^2\phi}{\partial\xi^2} = \omega_B^2\frac{m\kappa_0^2}{ck_z^2}\left(\frac{u-(\omega/k_z)}{\{[u-(\omega/k_z)]^2-(2e\phi/m\gamma^3)\}^{1/2}}-1\right) \tag{3.4}$$

for the case of an anisotropic single-axis dielectric. Here

$$\kappa^2 = k_z^2-\epsilon_0(\omega)\frac{\omega^2}{c^2} \quad \text{and} \quad \kappa_0^2 = k_z^2-\frac{\omega^2}{c^2}.$$

From (1.5) the boundary condition for the function ϕ is found:

$$\phi|_{r=R} = 0. \tag{3.5}$$

Taking into account that the nonlinear terms in the equations (3.3) and (3.4) are small and proportional to the beam density, the Bogoliubov–Krylov method (Kovtun and Rukhadze 1970) for solving these equations can be used. The boundary condition (3.5) and the results from the linear theory, obtained in § 2, suggest that the solution of equations (3.3) and (3.4) must be looked for in the following form:

$$\phi = \phi_{sl}J_l\left(\frac{\mu_{sl}}{R}\right)\cos\psi, \tag{3.6}$$

where $\psi = \omega\xi$. In this way, the beam-excited wave is assumed to be one-modal with fixed s and l. Thus the time and position dependences of fields are predetermined and only the amplitude ϕ_{sl} has to be obtained.

Further, the analysis will be confined to the nonlinear saturation of the fundamental axial symmetrical mode with $s = l = 0$ (ie $\mu_{00} = 2{\cdot}4$). As already shown, when the almost-longitudinal wave (when $\epsilon_0(\omega) < 1$) is excited by the beam, it is this mode that has the maximum growth rate. In the case of a transverse electromagnetic wave (at $\epsilon_0(\omega) > 1$), the favoured increase in this mode can be ensured by a preliminary modulation of the electron beam.

We shall begin the analysis with the case of an isotropic dielectric medium, in which the electron-beam excited waves are described by the equation (3.3). Substituting the solution (3.6) for the axial symmetrical modes ($s = l = 0$) in this equation and averaging on ψ the following equation for a longitudinal wave ($\epsilon_0(\omega) < 1$) is obtained:

$$\epsilon_0(\omega)k^2 J_0\left(\frac{\mu_{00}}{R}r\right)\phi_{00} = 2\frac{m\omega_B^2}{e}\begin{cases}[2(2-\eta_1^2)]^{1/2}\eta_1^2 C(\eta_1) \\ [2(2\eta_2^2-1)]^{1/2}K(\eta_2)\left(1-\dfrac{2E(\eta_2)}{K(\eta_2)}\right),\end{cases} \tag{3.7}$$

where $k^2 = \mu_{00}^2/R^2 + k_z^2$, $K(\eta_{1,2})$ and $E(\eta_{1,2})$ are elliptic functions,

$$C(\eta_{1,2}) = \eta_{1,2}^{-4}[(2-\eta_{1,2})K(\eta_{1,2}) - 2E(\eta_{1,2})]$$

and

$$\begin{aligned}\eta_1 &= \left(\frac{2e\phi_{00}\gamma^{-3}J_0((\mu_{00}/R)r)}{\frac{1}{2}m[u-(\omega/k_z)]^2 + e\gamma^{-3}\phi_{00}J_0((\mu_{00}/R)r)}\right) \\ \eta_2 &= \left(\frac{\frac{1}{2}m[u-(\omega/k_z)]^2 + e\gamma^{-3}\phi_{00}J_0((\mu_{00}/R)r)}{2e\phi_{00}\gamma^{-3}J_0((\mu_{00}/R)r)}\right).\end{aligned} \tag{3.8}$$

The upper expression on the right-hand side of equation (3.7) holds if

$$\tfrac{1}{2}m[u-(\omega/k_z)]^2 > e\phi_{00}\gamma^{-3}J_0((\mu_{00}/R)r)$$

and the lower expression holds if the inequality sign is reversed.

At small field amplitudes, when $\eta_1 \ll 1$, the dispersion equation of the linear theory, described in § 2, follows from equation (3.7). For large amplitudes, when

$$\tfrac{1}{2}m[u-(\omega/k_z)] \ll e\phi_{00}/\gamma^3,$$

the quantity η_2 in the entire range of variation of $r \leqslant R$ is in practice close to $1/\sqrt{2}$. Taking into consideration this finding, from equation (3.7), by integrating on r, we obtain

$$e\phi_{00} = m\gamma\left(\frac{\mu_{00}u|\delta|S(R)\omega_B^2}{5R^2\epsilon_0(\omega)\omega_0 k_0^2 J_1(\mu_{00})}\right), \tag{3.9}$$

where

$$k_0^2 = \frac{\mu_{00}^2}{R^2} + \frac{\omega_0^2}{u^2} \qquad \text{and} \qquad S(R) = \int_0^R J_0^{-1/2}\left(\frac{\mu_{00}}{R}r\right) r\,dr.$$

Now, considering that for longitudinal waves excited by the beam

$$\epsilon_0(\omega) = \epsilon_0(\omega_0) + \mathrm{i}\delta\partial\epsilon_0(\omega_0)/\partial\omega_0,$$

where ω_0 is the frequency spectrum (2.11) and δ is the growth rate (2.12), from (3.9) the threshold amplitude of a stationary longitudinal wave is finally obtained:

$$e\phi_{00} = m\gamma\left(\frac{3\mu_{00}uS(R)\omega_{\mathrm{B}}^2}{5R^2k_0^2\omega_0\mathrm{J}_1(\mu_{00})(\partial/\partial\omega_0)\epsilon_0(\omega_0)}\right)^{2/3}. \tag{3.10}$$

Similarly the stationary amplitude of a transverse electromagnetic wave, excited by the beam in an isotropic dielectric guide for which $\epsilon_0(\omega_0) > 1$, is found:

$$e\phi_{00} = \frac{m\gamma\mu_{00}u^2}{\omega_0^2}\left(\frac{3c^2S(R)\omega_{\mathrm{B}}^2}{5R^4\mathrm{J}_1(\mu_{00})\epsilon_0(\omega_0)(\partial/\partial\omega_0)\epsilon_0(\omega_0)\omega_0^2}\right)^{2/3}. \tag{3.11}$$

The quantities ω_0 and δ are thus determined by the relations (2.13) and (2.14).

Finally in case (ii) of an anisotropic dielectric, where the electron beam excites a wave with the spectrum (2.15) and (2.16), the threshold amplitude of a nonlinear wave is as follows:

$$e\phi_{00} = \frac{m\gamma\mu_{00}^2u^2}{\omega_0^2}\left(\frac{S(R)\omega_{\mathrm{B}}^2}{5R^4\mathrm{J}_1(\mu_{00})\epsilon_0(\omega_0)|\omega_0^2(\partial\epsilon_0(\omega_0)/\partial\omega_0)+(2\mu_{00}^2u^2/\omega_0R^2)\gamma^2(\gamma^2-1)|}\right). \tag{3.12}$$

4. Electromagnetic radiation flow near the the threshold of wave saturation

Now knowing the stationary amplitudes of electron-beam excited waves in a bounded dielectric medium, it is not difficult to calculate the vector of the electromagnetic radiation flow (Poynting's vector) and the conversion efficiency of the beam energy into energy of beam-excited radiation. In the case considered (excitation of an axial symmetrical mode), only the longitudinal component of period-averaged Poynting's vector is nonzero:

$$\mathscr{P}_z = \frac{c}{4\pi}\int_0^R r\,\mathrm{d}r\int_0^{2\pi}\mathrm{d}\varphi\overline{E_rB_\varphi} = \eta nmc^2(\gamma-1)u\pi R^2, \tag{4.1}$$

where η is the conversion efficiency of the kinetic energy flow of the beam into an electromagnetic radiation flow and the bar above E_rB_φ denotes time averaging. Further, the details of the calculation of the integral (4.1), which can be obtained easily using equations (2.9), (3.2) and (3.10)–(3.12), are omitted and only the final results for η are given.

In the case of an isotropic dielectric with $\epsilon_0(\omega_0) < 1$ (for instance, a plasma), when only the almost-longitudinal (potential) electromagnetic waves are excited by the electron beam, the following expression for the efficiency is obtained:

$$\eta_\parallel = \frac{g^{2/3}}{[1+(\mu_{00}^2u^2/R^2\omega_0^2)]^{5/3}}\frac{u^4\gamma}{2\omega_0^4c^2(\gamma-1)}\left(\frac{\mu_{00}^5\mathrm{J}_1(\mu_{00})S^2(R)\omega_{\mathrm{B}}^2}{25R^7(\partial/\partial\omega_0)\epsilon_0(\omega_0)}\right)^{2/3}. \tag{4.2}$$

When $\epsilon_0(\omega) > 1$ and the almost-transverse electromagnetic wave is excited by the beam, the expression

$$\eta_\perp = \frac{\mu_{00}^2\gamma^2}{2(\gamma-1)}\left(\frac{gc\mathrm{J}_1(\mu_{00})S^2(R)\omega_{\mathrm{B}}}{25R^3|\epsilon_0(\omega_0)|^{1/2}[(\partial/\partial\omega_0)\epsilon_0(\omega_0)\omega_0^2]^2}\right)^{2/3} \tag{4.3}$$

is obtained.

Finally, when an anisotropic single-axis waveguide is considered and the beam-excited wave is longitudinal–transverse, the expression

$$\eta=\frac{\mu_{00}^2u^4J_1^{2/3}(\mu_{00})}{2c^2R^2\omega_0^4}\frac{\gamma^6}{(\gamma-1)}\left(\frac{3\mu_{00}S(R)\omega_0^2\omega_B^{1/2}}{5R^2|\omega_0^2(\partial\epsilon_0(\omega_0)/\partial\omega_0)+(2\mu_{00}^2u^2/\omega_0R^2)\gamma^2(\gamma^2-1)|}\right)^{4/3} \tag{4.4}$$

is obtained for η.

It follows from the equations (4.2)–(4.4) that the conversion efficiency of the kinetic energy into an electromagnetic radiation flow, where an almost-longitudinal wave is excited, increases with the density as $n_0^{2/3}$ but when an almost-transverse wave is excited, it increases as $n_0^{1/3}$. At a small beam density the efficiency of an almost-transverse wave generation is much larger than the efficiency of the longitudinal-wave generation. Further, it is very important to notice that in an ultra-relativistic case ($\gamma \gg 1$) with the increase in the beam electron energy, the efficiency of electromagnetic wave generation in an anisotropic dielectric medium (for example in a magnetized plasma) is most quickly increased as $\gamma^{7/3}$. A simple evaluation shows that in the centimetre microwave range the efficiency of generation can be 10–20%.

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